Projection Reversibility Principle and Its application to Projection Corrections

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***Abstract*— Projection extends much more display possibilities beyond what traditional monitors can offer. However, strict condition requiring the projector directly facing a plane screen needs be satisfied; otherwise, distortion will occur. Distortion occurs in incline plane projection where the projector is not facing in normal direction and more complicated in the cases when the screen is a curved surface. To compensate, a rectified pattern is needed as a pre-processing before an intended image is projected to ensure no distortion. Existing solutions for the inclined planes involve warp perspective operation; solutions for curved surfaces are by approximating the curve surface as a collection of small inclined planes or grids, rendering the rectification for curved surface a repetitive rectification process on each of these constituent inclined plane. Alternatively, this study proposes an auto-calibration approach based on the reversibility principle of projection image. In addition to mathematical support for this approach, this study also presents an automatic calibration solution that together with projector and camera setup calibrates rectification patterns for both inclined planes and curved surfaces. The auto-calibration process requires no repetitive matrix manipulations which are prone to numerical errors and ensures no overlaps or gaps among approximated grids. To demonstrate, rectified projections to inclined wall, corner of ceilings, and cylinder surfaced buck are shown.**

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I. Projection Distortion

Augmented with innovative techniques developed in multimedia and virtual reality, people find more applications for projection. When multiple projectors work together, the range of projected view can be extended to general surfaces other than standard screens, like [1] and [2]. New interactive modes also emerges when projectors work with cameras that help interact people using the projector for presentation or get a better projective display for reality effects, like in.[3].and [4].

In its traditional application, when an image is projected onto a screen by a projector, a general perfect setting is needed to avoid distortion: the screen being plat and facing directly to the projector and also the observer is also facing in the same direction. When any combination of the conditions is violated, for instance, when the screen is curved, when the projector does not face perpendicularly to the screen; or when the observer is not facing in the same direction of the projector, then distortion will occur unless proper correction measure is employed. As shown in Fig. 1, given the screen being flat, when the observation direction is not parallel to that of the projection, a so called key stoned distortion in the form of an arbitrarily quadrilateral will result. A solution to this is by so called Projector-camera system [5], where an additional camera is used to observe the distortion caused by imperfect setting condition, and to correct accordingly. The distortion can be characterized by a Homography matrix H. To compensate, if the projection image is first corrected by the inverse effect of H^{-1} before projection, then a corrected undistorted image will be observed. In short, the H is to summarize the imperfect condition of the projection setting, and the inverse H^{-1} is to compensate such imperfect condition. For planar screen such H is easy to find. When the screen is curved, the screen can be approximated as a collection of multiple quadrilaterals, each of the quadrilateral being distorted by $H_{i,j}$, and the whole correcting measure is then reflected by the collection of the corresponding $H_{i,j}^{-1}$.

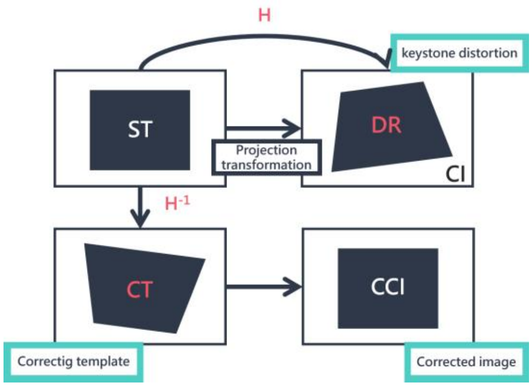

Fig. 1. Projection distortion and correcton

On the other hand, when the projection surface is not planar but curved. The correction process involved will be more complicate. Raskar et al. **[6]** propose a "two-pass rendering," which requires an additional camera first placed at where the observer is to observe the projected view. The first pass is to project a standard grid pattern like that of a checker board onto the curved surface; a distorted grid pattern is observed. The second pass is to embed the intended object image following the distorted pattern obtained in the first pass.

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This short note addresses the issue of finding a correcting template for imperfect projection condition such that the observer image is not distorted through a reversibility principle of projection image that allows approximately undistorted observed image when the projection condition is not perfect.

II. The Reversibility Principle of Projected Images

A. The Principle

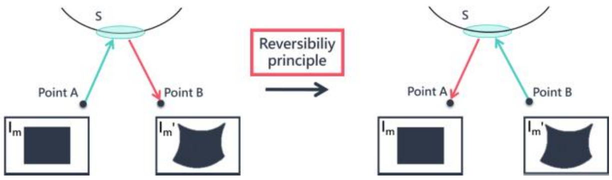

Fig. 2. Reversing projection direction resulting images unchanged

The reversibility principle can be stated with reference to the setup, shown in Fig. 2, which consists of a screen S, a projection point of A, and an observation point of B. When an image of $I_{AP} = I_m$ is projected from A onto the screen S, let the image observed by B be denoted by $I_{BO} = I_m{}'$. We now reverse the roles of projection and observation by situating the projector to point B, and the observer to point A. The reversibility principle states: if the image projected from point B onto the screen of S is $I_{BP} = I_m{}'$, then the observed image at A will be $I_{AO} = I_m$. Reversing refers to the role change of projection and observation. Reversibility refers to the fact that image source and projected image are reversed.

B. Application to Projection Correction

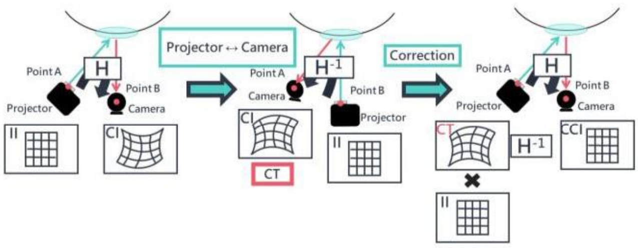

Fig. 3. Application of projection reversibility pinciple

The implication of reversibility principle in the context of projection correction can be stated as follows: Given a projection setting as depicted in the far left of Fig. 3, where the projection screen is a curved. By projecting a checker board image I_{st} from point A on the screen S, the image observed at point B can be expected to be distorted. The distortion can be attributed to the geometric relation reflected in the relative setting; we use a figurative notation of H to denote such distorting effect. To correct the distortion, we apply the reversibility principle to obtain the correction template CT. As shown in the middle of the Figure, the original observation point B is now placed with a projector, and the previous projection point A is now placed with a camera. A standard checker board of I_{st} is projected from B and the projected image captured by A the newly projected view, denoted by I_{AO}, which would be a different distortion of the checker board; we denoted it by CT, correction template. In effect, CT "corrects" the distortion caused by H, as such we use a figurative notation of H^{-1} to show the nullifying effect.

To see why this turns out to be a correcting template, we move to the far right of the Figure. When the CT is projected from point A onto the screen, by applying the reversibility principle, the image now observed at point B will be the projected image before the reversing, as shown in the middle of the Figure, which will be the standard checker board I_{st}, that is, the intended undistorted image.

Collectively, the H^{-1} can be approximated by a collection of $H_{i,j}^{-1}$, each of them corresponding to the distorted grid of (i,j) from its corresponding grid in standard checker board. If now, we are to project a general image I from point A, wishing to be observed without distortion at point B, we could approximately correcting the image by partitioning the whole image I into grids of checker board, $I_{i,j}$, each of them multiplied by the corresponding $H_{i,j}^{-1}$.

III. Explanation and Practical consideration

A. Explanation

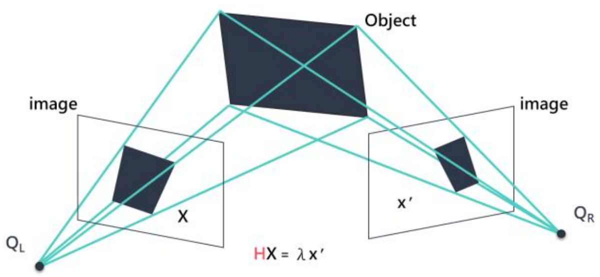

Fig. 4. Epipolar geometry

We coin this principle as have observed this reversibility phenomenon but to the best of our effort we could not find any citation for that. The underlying principle can be understood from both the epipolar geometry and pinhole camera model. First, with reference to Fig. 4, epipolar geometry is usually applied to explain the mapping relation between two camera images of a same three-dimensional view. The two camera images, taken from two different locations, can be summarized by a homography matrix. Second, the pinhole camera model relates the geometric relationship between a point in the three-dimensional space and its project onto the image plane. In fact, the same model can be applied to projector as well with the direction of light goes reversely: from a point in the projection image plane onto a point in the projection screen. Now, with reference to Fig. Y, suppose the mapping between the project image to the observed image is summarized by H, the by reversing the direction of light with the roles of projector and camera interchanged, the mapping between the new project image to that of the new observed image will be summarized by H^{-1}.

B. Practical Consideration

In reality, it is difficult to implement and empirically prove the reversibility principle. The reversal of light direction is implemented by interchanging the projector and the camera involved. With this interchange, two issues have to be considered:

(1) The position in the three-dimensional space and the optical axis direction need to be the same before and after the replacement.

(2) Parameters like the optical focus and the resolutions of the image size for both projector and camera may not be the same.

As a result, the observed image by the replaced camera cannot be used directly by the projector right after the replacement. In addition, it has to be considered that when the projection surface is curved, a checker board of denser grids is recommended for better approximation and thus the correction result.

To accommodate the first fact, we, alternatively, stack the camera and the replacing projector (and vice versa) together as a binding unit, without really replacing one another. The differences in optical parameter and resolution are modified in the associated transformation, as shown in Fig. 5.

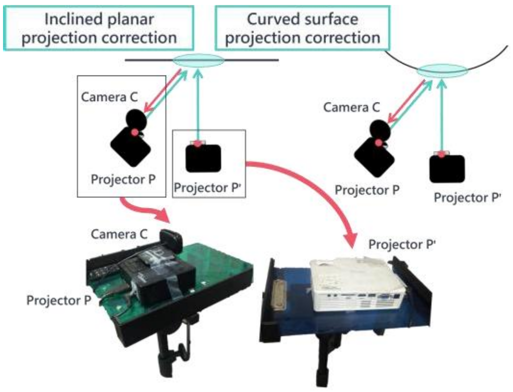

Fig. 5. Stacking projector and replacing camera together

Accommodation for the second issue is more involed. For the interest of limited presentation space, Fig. 6 depicts the process when correcting a curved surface is involved: the projection effect for the particular seetting of the grid is first compute by $H_{\text{homography}}$, followed by a translated scale transformation H_{scale}.

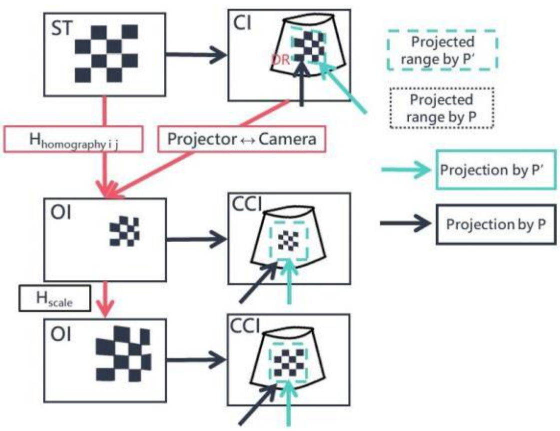

Fig. 6. Obtaining correction template for curved projection surface

For the case of inclined planar screen, basically four corners of the entire check board are sufficient to decide the involved H whereas, when the projection is a curved surface, each distorted grid representing each approximation of the projected grid needs to be marked by the associated four corners. In parallel or one in one, mapping for all the grids in the entire checker board are to be found through lines and intersections of lines from image processing.

IV. Experimental Results and Remark

We have implemented a system integrating generating a correction template and compositing the corrected image under a given projection setup environment. The corrected projects have been conducted when the projection surfaces are: inclined plane, cylinder bucket, and ceiling corner joined by two perpendicular sides of walls.

A. The integrated system

The whole automatic process for projection correction on curved surface is shown in the flowchart of Fig. A. A standard checker board, ST, is projected from the replaced projector, resulting in a distorted checker board observed by the replaced camera. This distorted checker board image from its standard counterpart is then used as a correction template, CT. Each of the distorted grid, marked by the four corners, represents the transformation $H_{i,j}^{-1}$ due to the particular projection setting for the corresponding standard grid. As a whole, the correcting template is a collection of $H_{i,j}^{-1}$ for all approximated grids. Once CT is obtained, an intended projected image, II, can then be corrected before the real projection takes place by the projector. In doing so, the II is partitioned accordingly as the grids in ST, and each of the partitioned $\mathrm{II}_{i,j^{-1}}$ is multiplied by $H_{i,j}^{-1}$ before projection.

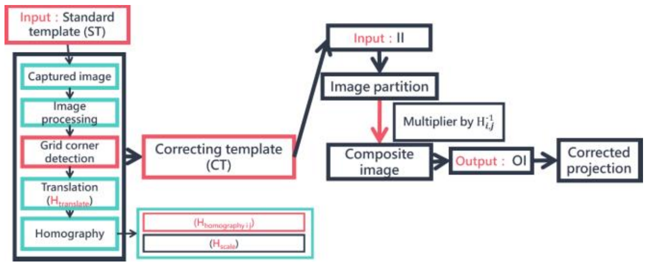

Fig. 7. Automatic projection correction

B. Corrected results

Three types of surfaces have been experimented: inclined screen, cylinder bucket, and ceiling corner joined by two side of wall, as depicted from Fig. 8 through Fig.10 ,spectively, wherein, panels from left to right are the settings, un-corrected (distorted) images, and the corrected images.

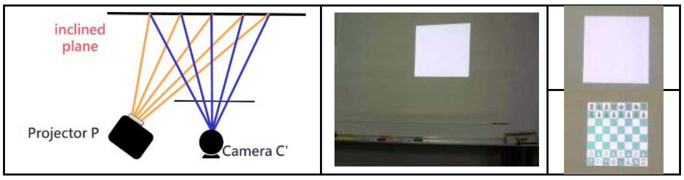

Fig. 8. Correction for Inclined plane projection

Fig. 9. Correction for curved bucket projection

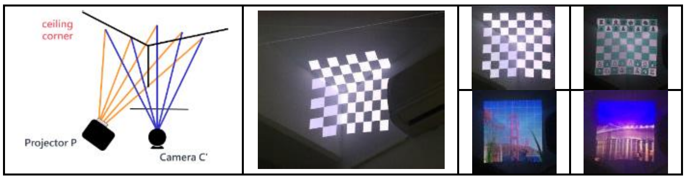

Fig. 10. Correction for ceiling corner projection

C. *Remarks*

Interested readers are referred to [7] for details of the proposed technique of applying the reversibility principle to projection correction. Preliminary work on how this technique can be applied to elimination of blind spots is reported in [8]. In addition, the technique presented in this paper can also be extended to applications in interactive virtual reality where projected view has to be adjusted based on the viewer's position.

ACKNOWLEDGMENT

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