Proposal of Sub-optimum Algorithm for Trellis-based SLM Reducing PAPR of FBMC-OQAM Signals

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***Abstract*— Filter bank multi-carrier with offset quadrature amplitude modulation (FBMC-OQAM) has several advantages. FBMC-OQAM is considered a major candidate for the next generation mobile communication systems. The high peak-to-average power ratio (PAPR) is one of disadvantage for FBMC-OQAM signal which could be solved by the Trellis-based SLM (TSLM) technique. This TSLM technique has a much better PAPR reduction performance. However, the trellis-based algorithm has high computational complexity. This paper proposes the sub-optimum algorithm for trellis-based SLM scheme. The proposed algorithm show much lower computational complexity than trellis-based SLM scheme with small degradation of PAPR reduction performance. On the other hand, the PAPR reduction performance of proposed algorithm shows better than conventional SLM with similar computational complexity when the number of L symbols is large. The efficiencies of proposed scheme were verified by using various computer simulations.**

***Keywords*— FBMC-OQAM, PAPR, SLM and Trellis-based**

I. INTRODUCTION

Filter Bank Multi-Carrier with Offset Quadrature Amplitude Modulation (FBMC-OQAM)[1-3] is considered a candidate radio waveform for the next generation of a mobile standard. FBMC-OQAM system provides higher data rate because FBMC-OQAM does not require the cyclic prefix CP, excellent frequency localization in addition to very low side lobes in its power spectral density (PSD), robustness to phase noise and frequency offsets. This FBMC-OQAM modulation scheme can achieve orthogonality between the adjacent sub-channels which leads to less adjacent channel interference. The drawback of FBMC-OQAM has the large peak in the time domain signal as OFDM signal.

Many PAPR reduction schemes have been proposed for both OFDM system and FBMC-OQAM system such as Selected Mapping (SLM)[4], Tone Reservation (TR) and Partial Transmit Sequence (PTS)[5] and so on. However, FBMC-OQAM signals have a very different signal structure when compared with OFDM signal that has the adjacent data blocks overlap with one another. The PAPR reduction schemes based on OFDM could not directly apply to FBMC-OQAM. The past symbols of FBMC-OQAM signal have to consider when the optimum PAPR reduction is calculated on the current symbol. The Dispersive SLM have been proposed to overcome this problem. The determined optimum of phase pattern for a conventional scheme on the current symbol is not efficient for Dispersive SLM scheme. The trellis-based algorithm can show the better optimum path of phase pattern when compared with the conventional determination of phase pattern.

To overcome the computational complexity problem, we propose a sub-optimum algorithm for trellis-based SLM reducing PAPR of FBMC-OQAM signals. The proposed scheme only applies the trellis-based algorithm for the first and the second symbols. After the first two symbols, we apply the conventional least PAPR algorithm in an advance $m+1^{\text{th}}$ symbol and apply the phase pattern of current m^{th} symbols by using the phase pattern of the previous symbol. The proposed algorithm has the potential to enhance the performance of reducing high computation complexity, improving PAPR reduction performance. The computational complexity of each scheme can be formulated by flowing equation in table 1.

II. SYSTEM MODEL

Figure 1 shows the transmitter model of FBMC-OQAM system with PAPR reduction scheme. The proposed scheme will refer to this transmitter model for the detailed explanation. The PAPR reduction scheme is additional processing for reducing the peak signal of FBMC-OQAM at the transmitter side. The input data (X) is randomly complex number as shown by the following equation,

$$X_{m,n} = R_{m,n} + j.I_{m,n},\quad 0 \le n \le N-1,\quad 0 \le m \le M-1 \qquad (1)$$

where, $R_{m,n}$ and $I_{m,n}$ are the real and imaginary parts of the input data over $m-th$ symbol and $n-th$ sub- carrier,

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respectively. A number of the symbols is defined by M. A number of the sub-carriers is defined by N. The OQAM modulation, real and imaginary symbols are transmitted at an interval $T/2$. A period of an FBMC- OQAM symbol is transmitted by T. The input data (X) in equation one at $m-th$ can be grouped as a vector, denoted by Xm that

$$X_m = \left(X_{m,0}, X_{m,1}, \ldots X_{m,N-1}\right)^{\mathrm{T}}, \quad 0 \le m \le M-1 \tag{2}$$

where, the transpose operation is defined by $[\ ]^T$. The modulated OQAM signal of the complex symbol vectors $\{X_m\}_{m\ 0}^{M-1}$ into real symbol can be defined by $\{a_{m',n}\}_{m'=0}^{2M-1}$. The transmitted signal of the conventional FBMC signal can be formulated as the following equation,

$$s(t) = \sum_{m'=0}^{2M-1} \sum_{n=0}^{N-1} a_{m',n} h(t - m'T/2) e^{j\frac{2\pi}{T}nt} e^{j\varphi_{m'n}} \tag{3}$$

where, the impulse response of the prototype filter is defined by $h(t)$, a real symbol mapping from the complex $X_{m,n}$ is defined by $a_{m',n}$ with m varying from 0 to $M-1$ as shown below,

$$m' = \begin{cases} 2m & m' \text{is even} \\ 2m+1 & m' \text{is odd} \end{cases} \tag{4}$$

$$a_{m',\mathrm{n}} \quad \begin{cases} (1-\rho).R_m^n + \rho.I_m^n & m' \text{is even} \\ \rho.R_m^n + (1-\rho).I_m^n & m' \text{is odd} \end{cases} \tag{5}$$

where $\rho \in \{0,1\}$ is defined as n modulo 2. The mathematical significance of this time-staggering rule appears in [5] that has got a phase term $\varphi_{m',n}$, which is set to be $\frac{\pi}{2}(m'+n) - \pi m'n$ without loss of generality. In this paper, we apply the PHYDYAS filter as the prototype filter. This prototype filter is designed based on frequency sampling technique. The analytical parameters implicated in this filter design are the number of sub-carriers N, the overlapping factor of the prototype filter is K, the roll-off and the length of the filter are $L \quad KN$ with desired values $F(k/L)$, where $k = 0,1,2 \ldots L-1$

$$F_0 = 1,\ F_1 = 0.97196,\ F_2 = 1/\sqrt{2},\ F_3 = \sqrt{1-F_1^2}, \tag{6}$$
$$F_l \quad 0, \quad 4 < l < L-1$$

The impulse response of the PHYDYAS filter can be expressed as the following equation,

$$h(t) \quad \begin{cases} \frac{1}{\sqrt{A}}[1 + 2\sum_{l\ 1}^{K-1}(-1)^l F_k \cos\left(\frac{2\pi l t}{KT}\right)] & t \in [0, KT] \\ 0 & elsewhere \end{cases} \tag{7}$$

where the normalization constant, $A = KT\left[1 + 2\sum_{l\ 1}^{K-1} F_l^2\right]$ and $K = 4$.

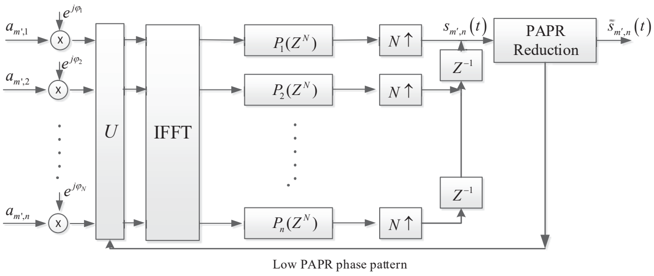

Figure 1. Transmitter model of FBMC-OQAM system with PAPR reduction scheme.

A continuous time base-band signal $s(t)$ in Eq.(3) that is transmitted during a symbol period T. The PAPR performance of FBMC signal that includes summation of overlap signal over a symbol period T is calculated by the following equation as below,

$$PAPR_{s(t)} \quad \frac{\max_{0 \le t \le T} |s(t)|^2}{\frac{1}{T}\int_0^T |s(t)|^2 .dt} \tag{8}$$

III. PAPR REDUCTION BY USING SLM SCHEME

. This section describes a detail of the basic PAPR reduction for the conventional SLM scheme and additional algorithms for PAPR reduction improvement in the FBMC signal such as Dispersive SLM algorithm (DSLM) and Trellis-based SLM algorithm. At the end of this section, describes the detailed proposal of sub-optimum for trellis-based SLM scheme.

A. Conventional SLM algorithm

Firstly, the phase pattern is generated by U complex phase rotation vectors $\phi^{(\mathrm{u})}$, for $0 \le u \le U-1$, of length N as,

$$\phi^{(\mathrm{u})} \quad \begin{cases} (1, \ldots \quad)^T & u = 0 \\ \left(\phi_0^{(\mathrm{u})}, \ldots, \phi_{N-1}\right)^T, & 1 \le u \le U-1 \end{cases} \tag{9}$$

where $\phi_k^{(\mathrm{u})}$ is the phase pattern $k-th$ element of $\phi_k^{(\mathrm{u})}$ is defined as,

$$\phi_k^{(\mathrm{u})} \quad e^{j\psi_k}, \qquad 0 \le u \le U-1, 0 \le k \le N-1 \tag{10}$$

where ψ is a uniformly distributed phase between 0 and 2π,

$$X_m^{(\mathrm{u})} \quad X_m \circ \phi^{(\mathrm{u})}, \qquad 0 \le u \le U-1, \quad 0 \le m \le M-1 \tag{11}$$

where $\circ$ is element-by-element multiplication. $X_m^{(\mathrm{u})}$ keeps the same information and identical constellation as X. The time domain signal $x^{(\mathrm{u})}$ is obtained by applying inverse discrete Fourier transform operation to $X_m^{(\mathrm{u})}$. The minimum PAPR can be achieved by optimum phase pattern and can be written as equation (8). $\tilde{s}_{m',\mathrm{n}}(t)$ are the transmitted signal with least PAPR after passed the PAPR reduction block as shown in figure 1.

B. Dispersive SLM algorithm

In the Dispersive SLM scheme, each input symbol vector X_m in (2) is phase rotated with U different input vectors giving $X_m^{(\mathrm{u})}$. The symbols $a_{m',n}^{(\mathrm{u})}$ are chosen from $X_m^{(\mathrm{u})}$ as conventional SLM scheme. However, the optimal rotation for a given symbol, the overlapping of its previous symbols are also calculated for the current PAPR. Since, the PHYDYAS prototype filter impulse response is taken by 4*T*. We consider the overlap of previous symbols for a given *m*-th input symbol as shown in the following equation,

$$\tilde{s}_{m',\mathrm{n}}(t)=\underbrace{\sum_{m'=0}^{2M-1}\sum_{n=0}^{N-1}a_{m',n}^{(\mathrm{u}_{\min}^{p})}h\left(t-m'T/2\right)e^{j\frac{2\pi}{T}nt}e^{j\varphi_{m'n}}}_{\textit{overlappings pass symbols}}+\underbrace{\sum_{m'=2m}^{2M-1}\sum_{n=0}^{N-1}a_{m',n}^{(\mathrm{u})}h\left(t-m'T/2\right)e^{j\frac{2\pi}{T}nt}e^{j\varphi_{m'n}}}_{\textit{current symbol}} \tag{13}$$

where $a_{m',n}^{(\mathrm{u}_{\min}^{p})}$ are from previously selected symbols $X_m^{(\mathrm{u}_{\min}^{p})}$, $0\le m\le M-1$ and $0\le u\le U-1$. The optimally rotated symbol is selected among others based on least PAPR criterion.

C. Trellis-based SLM algorithm

If the M input symbol vectors are transmitted by $\left\{X_0^{(u)},X_1^{(u)},X_2^{(u)}\dots X_{M-1}^{(u)}\right\}$ then, we need to find $\phi^{(u)}$, which is the optimal phase pattern of *M* different phase rotation vectors that provides the least PAPR.

$$\Phi=\left\{\phi^{(u_{min}^{0})},\phi^{(u_{min}^{1})},\phi^{(u_{min}^{2})}\dots,\phi^{(u_{min}^{M\text{-}1})}\right\} \tag{14}$$

where, $\phi^{(u_{min}^{0})},\phi^{(u_{min}^{1})},\phi^{(u_{min}^{2})}\dots,\phi^{(u_{min}^{M\text{-}1})}$ are the indices of optimal phase rotation vectors, which the input symbol vectors equal M symbols and it needs to be sent to the receiver as side information. With M input symbols and U phase rotation vectors, we need to find the best path as the trellis of Fig. 2 that provides the lowest PAPR performance. The optimal path in the trellis-based SLM algorithm means finding the multiplicative vectors with least PAPR by solving equation (13), with employ a trellis diagram as shown as below.

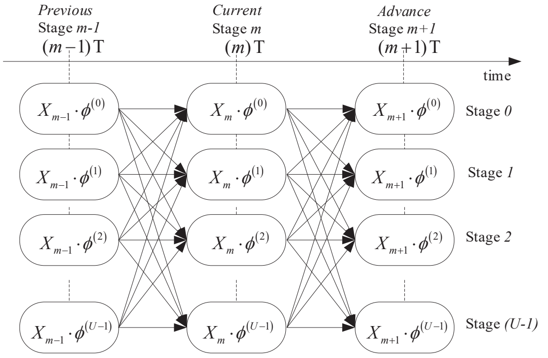

Figure 2. Example of the trellis diagram between 3 stages with 3 states.

For FBMC-OQAM symbol $s_{m',\mathrm{n}}(t)$ every *m*-th, $0\le m\le M-1$ are obtained by modulation of input symbol vector X_m ,which is represented as the *m*-th stage in the trellis. The modulated signal X_m will reach at time instant mT. At each stage, there will apply *U* to the different states, representing the rotated FBMC-OQAM symbols. The *i*-th trellis state is among states of rotation as phase rotation vector $\phi^{(i)}$. Between every two stages, there is U^2 a number of possible paths. The input symbol vectors $X_m^{(u)}$ and $X_{(m+1)}^{(u')}$ are FBMC symbol of *m*-th and (*m*+1)-th respectively, which is represented by the selected path by trellis algorithm $\zeta_{(m\Rightarrow m+1)}^{(u,u')}$ between u'-th state in *m*-th stage and u''-th state in (*m*+1)-th stage, where ⇒ represents a transition between two successive stages. The detail of trellis-based algorithm are explained by the following Ref. [5].

Form the figure 2, it's can be observed that it is not usable for practical hardware because high calculation complexity when the large number of subcarriers. To overcome this problem, the proposal of sub-optimum TSLM algorithm will be described below.

D. Proposal of sub-optimum TSLM algorithm

The proposed scheme had been modified based on the trellis algorithm as previously mentioned. The main modification is separated by 1. First and second symbols employ the trellis algorithm. Then, 2 the optimum phase rotation vector on (*m+1*)-th is kept in the memory, which will employ in the *m*-th symbol at the next iteration. The second steps will be repeated until the last symbols (*M-1*). The dispersive algorithm also employed to avoid the regeneration of high peak from the overlap symbols. The detail of proposed scheme can be drawn in the block diagram as shown in figure 3.

TABLE I. COMPUTATIONAL COMPLEXITY COMPARISON

Algorithm	Complexity
Conventional SLM	$[MU]$
Dispersive SLM	$[MU]$
Trellis-based SLM	$[(M-1)U^2]$
Proposed algorithm based TSLM	$[U^2+(M-2)U]$

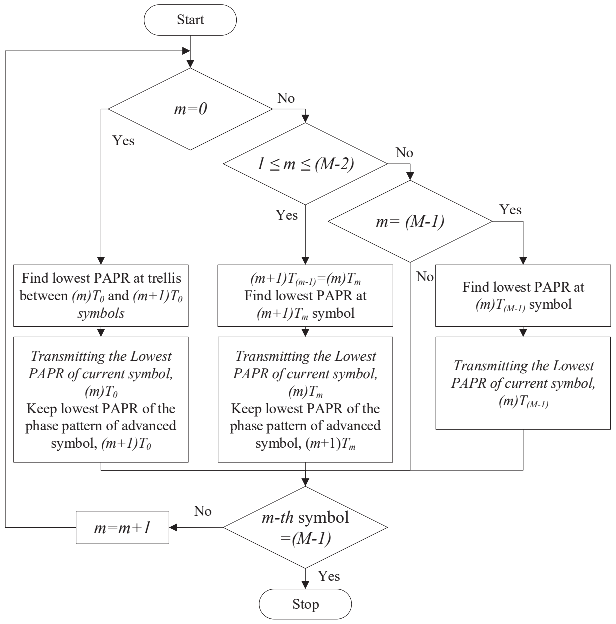

Figure 3. Flowchart of sub-optimum TSLM algorithm.

IV. PERFORMANCE EVALUATIONS

This section presents the performance verification of proposed scheme by using the various computer simulation. The simulation parameters can be listed in table below,

TABLE II. SIMULATION PARAMETERS

Schemes	FBMC-OQAM
Modulation	16OQAM
Demodulation	Coherent
Allocated bandwidth	5MHz
Number of FFT points(N)	256
Number of sub-carriers(M)	64
Number of clusters (V)	4
Number of discrete phases (W)	4
Overlapping factor(K)	4
Number of Phase Pattern (U)	16
Prototype filter	Phydyas Filter (PF)

Figure 4, the modulation technique is 16 OQAM. The number of subcarriers (N) is 512. The number of phase pattern(U) is 16. We have to apply the dispersive algorithm for all the PAPR reduction scheme for equity comparison because the passed symbol will regenerate the high peak again. The PAPR performance of FBCM-OQAM with PAPR reduction schemes shows lower PAPR performance when compared with the conventional FBMC signal. The figure 4 that we can observe the CCDF of PAPR performance at 10^{-2} and 10^{-3}, which is shown the best PAPR performance by using trellis-base SLM and proposed scheme respectively. The proposed scheme clearly shows the improved PAPR performance when compared with conventional PTS scheme.

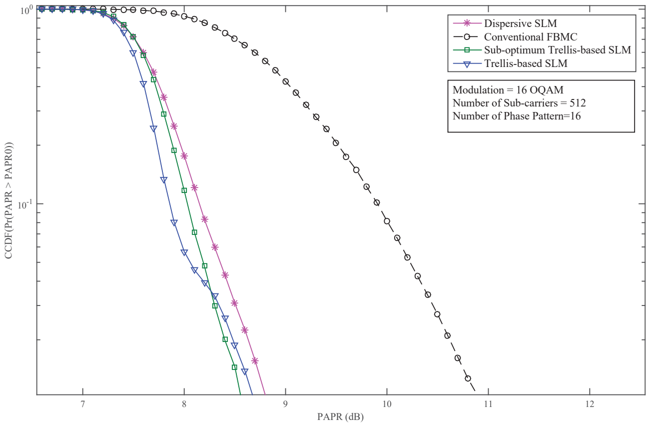

Figure 4. Comparison PAPR performance between conventional and proposed scheme for FBMC signal.

V. CONCLUSIONS

In this paper, we proposed the new searching algorithm for determining the best phase rotation vectors with low PAPR and less computation complexity then trellis-based SLM algorithm. We confirm that the average PAPR performance of the proposed scheme is similar to TSLM scheme. However, the computation complexity of proposed scheme is much lower than TSLM scheme. Assume, the number of symbols (M) is 20. The complexity ratio between proposed scheme and TSLM scheme is (544/4864=11.18). As a result, the compaction complexity can be reduced up to around 89% of the conventional scheme by using the proposed scheme. On the other hand, the simulation result shows that the proposed scheme has better PAPR reduction performance when compared with Dispersive SLM with a little additional of computation complexity. The proposed scheme has no additional modification of FBMC-OQAM transmitter.

REFERENCES

[1] Fa-Long Luo (Editor), Charlie (Jianzhong) Zhang (Editor), "Signal Processing for 5G: Algorithms and Implementations, " Wiley-IEEE Press, August 2016.

[2] P. Banelli, S. Buzzi, G. Colavolpe, A. Modenini, F. Rusek and A. Ugolini, "Modulation Formats and Waveforms for 5G Networks: Who Will Be the Heir of OFDM?: An overview of alternative modulation schemes for improved spectral efficiency," IEEE Signal Processing Magazine, vol. 31, no. 6, pp. 80-93, Nov. 2014.

[3] P. Siohan, C. Siclet and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank theory," in IEEE Transactions on Signal Processing, vol. 50, no. 5, pp. 1170-1183, May 2002.

[4] K. C. Bulusu, H. Shaiek, D. Roviras, and R. Zayani, "PAPR Reduction for FBMC-OQAM Systems using Dispersive SLM Technique," 11th IEEE International Symposium on Wireless Communication Systems, pp. 568-572, Aug. 2014.

[5] S. S. K. C. Bulusu, H. Shaiek and D. Roviras, "Potency of trellis-based SLM over symbol-by-symbol approach in reducing PAPR for FBMC-OQAM signals," 2015 IEEE International Conference on Communications (ICC), London, pp. 4757-4762, 2015.